Math 10

Lesson 1-3 Squares, Cubes and Roots

I. Perfect squares and square roots

Any number multiplied by itself results in a perfect square.

For example
- 3 x 3 = 9
- 5 x 5 = 25
- 16 x 16 = 256

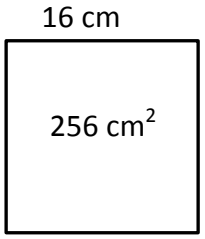
} 9, 25 and 256 are the perfect squares of 3, 5 and 16

The square root of a given number is a number which, when multiplied by itself, results in the given number.

For example
- $\sqrt{9} = 3$
- $\sqrt{25} = 5$
- $\sqrt{49} = 7$

} The square roots of 9, 25 and 49 are 3, 5 and 7

Another way to understand perfect squares and square roots is geometrically. A square is a rectangle with equal sides. For example, 256 is a perfect square.



$\therefore$ 256 is a perfect square and 16 is its square root

Question 1

Write the first 12 perfect squares and their square roots (i.e. $2^2 = 4$ and $\sqrt{4} = 2$)

Example 1 Find $\sqrt{576}$ using estimation or prime factorization .

Estimation

Consider that $20^2 = 400$ and $25^2 = 625$. 576 is less than 625. Therefore, try 24·24 and we get 576.

$\therefore \sqrt{576} = 24$

Prime factorization

The prime factorization of 576 is

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

Since we are finding the square root, split the factors into two equal groups. (If we were looking for the fourth root, we would arrange the factors into four equal groups.)

$= (2 \cdot 2 \cdot 2 \cdot 3) \cdot (2 \cdot 2 \cdot 2 \cdot 3)$

$= 24 \cdot 24$

$\therefore \sqrt{576} = 24$

Question 2

Find $\sqrt{1296}$ using prime factorization and/or estimation.

II. Perfect cubes and cube roots

Any number that can be represented as the volume of a cube with a <u>whole number</u> side length is a <u>perfect cube</u>.

OR

A <u>perfect cube</u> is any number that is the product of three identical factors.

For example
- 3 x 3 x 3 = 27
- 5 x 5 x 5 = 125
- 16 x 16 x 16 = 4096

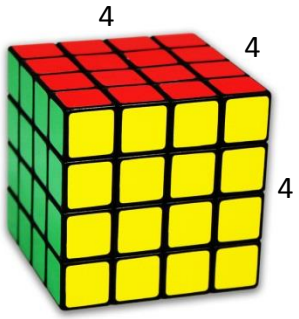
} 27, 125 and 4096 are the perfect cubes of 3, 5 and 16

The <u>cube root</u> of a given number is the number which, when multiplied by itself three times, results in the given number.

For example
- $\sqrt[3]{27} = 3$
- $\sqrt[3]{64} = 4$
- $\sqrt[3]{343} = 7$

} The cube roots of 27, 64 and 343 are 3, 4 and 7

Geometrically, a cube is a rectangular prism with equal edges. For example, 64 is a perfect cube.



$\therefore$ 64 is a perfect cube and 4 is its cube root

Question 3

Write the first 10 perfect cubes and their cube roots (i.e. $2^3 = 8$ and $\sqrt[3]{8} = 2$).

Example 2 Find $\sqrt[3]{2744}$ using estimation or prime factorization .

Estimation

Consider that $10^3 = 1000$ and $20^3 = 8000$. 2744 is larger than 1000 and smaller than 8000. Using guess and check

Try 13: $13^3 = 2197$ (small but close)

Try 14: $14^3 = 2744$

$\therefore \sqrt[3]{2744} = 14$

Prime factorization

The prime factorization of 2744 is

$= 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7$

Since we are finding the cube root, split the factors into three equal groups.

$= (2 \cdot 7) \cdot (2 \cdot 7) \cdot (2 \cdot 7)$

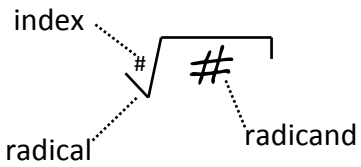
$= 14 \cdot 14 \cdot 14$

$\therefore \sqrt[3]{2744} = 14$

Question 4

Find $\sqrt[3]{1728}$ using prime factorization and/or estimation.

III. The radical symbol



The symbol is called a radical. (Who would think of such a word? See ** below.) The number of interest is called the radicand and the root we are looking for (2^{nd}, 3^{rd}, 4^{th}, 5^{th}, etc.) is called the index.

For example, the 5^{th} root of 789 would be written as $\sqrt[5]{589}$.

**According to Florian Cajori in *A History of Mathematical Notation* the word "radix" was used for square root in the thirteenth century or so, and was abbreviated as R_{x}. The symbol that looks like a check √ (radical sign without the "roof") originated in Germany, in the 1500's. It started out looking like a musical note. If you had a long expression under the radical sign, the expression was put in parentheses, and later, placed with a line over it. This is where the current symbol came from. Rene Descartes in his *La Geometrie* (1637) seems to be the first to place the line on top for grouping. So, no one really "invented" the sign - it developed over the years. But if you need a specific person, Descartes seems to be the one to first use the present day version of the symbol. (By: Doctor Bombelli, The Math Forum at http://mathforum.org/dr.math/)

Question 5

What are the index and radicand for each of the following:

$\sqrt[3]{4}$ $\sqrt[4]{3}$ $\sqrt{5}$

Example 3 Estimate the cube root of 10 to one decimal place .

Consider that $2^3 = 8$ and $3^3 = 27$. Since 10 is closer to 8, let's try 2.1 and 2.2

$2.1^3 = 9.26$

$2.2^3 = 10.65$

Since 10 is closer to 10.65, $\sqrt[3]{10} \cong 2.2$

Question 6

Estimate the following to at least 1 decimal place:

(a) the square root of 81

(b) the square root of 52

(c) the cube root of 64

(d) the cube root of 52

(e) the fourth root of 52

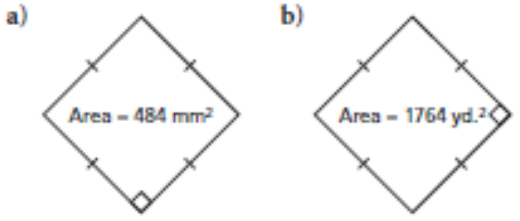
IV. Assignment

1. Determine the square root of each number. Explain the process used.
   a) 196 d) 289

2. Determine the cube root of each number. Explain the process used.
   a) 343 d) 1331

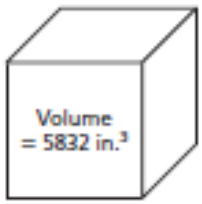
3. Use factoring to determine whether each number is a perfect square, a perfect cube, or neither.
   a) 225 b) 729 c) 1944
   d) 1444 e) 4096 f) 13,824

4. Determine the side length of each square.

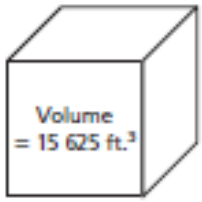


5. Determine the edge length of each cube.

a)



b)



6. In February 2003, the Battleford's Chamber of Commerce in Saskatchewan placed a cage containing a 64-cubic foot ice cube along Yellowhead Highway. Local customers were asked to predict when the ice cube would melt enough for a ball above the ice cube to fall through it. What was the surface area of the cube?

7. A cube has surface area 6534 square feet. What is its volume?

8. Is it possible to construct a cube with 2000 interlocking cubes? Justify your answer.

9. Write 3 numbers that are both perfect squares and perfect cubes.

10. During the Festival du Voyageur in Winnipeg, Manitoba, teams compete in a snow sculpture competition. Each team begins with a 1440-cubic foot rectangular prism of snow. The prism has a square cross-section and height 10 ft. What are its length and width?

11. Determine the dimensions of a cube for which its surface area is numerically the same as its volume.

12.
    a) Determine the side length of a square with area $121x^4y^2$.
    b) Determine the edge length of a cube with volume $64x^6y^3$.

13. Which pairs of perfect cubes have a sum of 1729?

14. A cube has a volume of 2197 m^3. Its surface is to be painted. Each can of paint covers about 40 m^2. How many cans of paint are needed? Justify your answer.